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Measuring static viscous permeability of porous absorbing materials.

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Conventional acoustical methods for measuring the permeability or flow resistivity of a porous material require a priori estimation of the porosity. In this work, an acoustical method is presented, in which a simplified expression (independent of both the frequency and porosity) for the transmitted waves at the Darcy's regime (low frequency range) is derived, and used for the inverse determination of both the viscous static permeability (or flow resistivity) and the thickness of air-saturated porous materials. The inverse problem is solved based on the least-square numerical method using experimental transmitted waves in time domain. Tests are performed using industrial plastic foams. Experimental and numerical validation results of this method are presented, which show the advantage of measuring the viscous permeability and thickness of a porous slab, without the required prior knowledge of the porosity, but by simply using the transmitted waves.

I. INTRODUCTION

The static viscous permeability k_0 is one of the most important parameters, which appears in the theories of sound propagation in porous media, in the low frequency regime^{1–5}. This parameter is related to the specific flow resistivity σ by the relation : $k_0 = \eta/\sigma$, where η is the fluid viscosity. Several methods^{6–23} have been developed in the past to measure the static viscous permeability or the flow resistivity. Among these methods, we distinguish between the so-called direct methods^{6–11} which do not use sound waves, and indirect methods^{12–23} that use sound waves transmitted or reflected by the porous material. The practical implementation of the direct methods could be both complex and expensive. Most of the acoustic (indirect) methods^{6–23} require a priori estimation of the porosity, or other non acoustic parameters^{1–3} (tortuosity, viscous and thermal characteristic lengths, thermal permeability). The proposed procedure is an indirect acoustical method for measuring the static viscous permeability (and therefore flow resistivity), without knowing in advance the porosity or other non-acoustic setting. This method improves and simplifies the approach developed in our previous work^{18–20}. The originality of this contribution with respect to Refs. 18-20 is the use of a simplified transmission coefficient which is independent of the frequency and porosity. Some techniques use impedance tube, in which standing waves are generated, and where two^{12–16} or three¹⁷ microphones are used for experimental measurements. In this case, a calibration of the microphones is necessary for a good quality of the results. In our proposed method, a tube is used, in which transient sound waves propagate. A single microphone^{18–20} is used for the measurement of experimental signals, therefore, no calibration is required.

Sound propagation in air-saturated porous material is described by various physical parameters, which are different according to the frequency domain¹. The high and low frequency ranges^{1–3}, are defined by comparing the viscous and thermal skin thicknesses $\delta = (2\eta/\omega\rho)^{1/2}$ and $\delta' = (2\eta/\omega\rho P_r)^{1/2}$ with the radius of the pores r (ρ is the density of the saturating fluid ; ω the

pulsation frequency ; P_r the Prandtl number). In the asymptotic domain (high frequencies), the skin thicknesses become narrower and the viscous effects are concentrated in a small volume near the frame $\delta \ll r$ and $\delta' \ll r$. The fluid-structure interactions are described by the tortuosity¹, viscous² and thermal characteristic lengths³. In the viscous domain (low frequencies)³, the skin thicknesses δ and δ' are much larger than the radius of the pores. The main important parameters in this frequency domain are ; the static viscous and thermal permeabilities³ and viscous and thermal tortuosities³. In addition to these parameters, the porosity ϕ is a key parameter playing an important role for all frequencies. In the Darcy's regime^{18–20,24,25} (very low frequencies), the static viscous permeability is the most influential parameter ; the propagation equation reduces to a diffusion equation^{7–9}. The acoustic wave does not propagate, but is just attenuated. The direct and inverse problems were solved in time domain^{7–9}, using reflected⁷ and transmitted^{8,9} experimental data, thus obtaining a good estimate of the static viscous permeability (or specific flow resistivity), knowing in advance, the value of the porosity.

In this work, we present an improved method to determine simultaneously the static viscous permeability (or flow resistivity) and thickness of the porous material, without knowing in advance, any other non acoustic parameter. The interest of solving the inverse problem with respect to the thickness of the material is to verify the results of the inversion, since the thickness of the material is easily measurable. The inverse problem is solved using experimental transmitted waves. We derive a simplified expression for the transmission coefficient in the Darcy's regime. The obtained expression is independent of frequency and porosity. The viscous static permeability and thickness of the material are the only parameters involved. The transmitted waves predicted by the simplified expression of the transmission coefficient coincide with those obtained using the classical expression^{19,20}. Inversions on real experimental data are made, using waves transmitted by samples of air saturated porous foams. The inversion results are satisfactory and open new perspectives for the characterization of air-saturated porous materials.

II. EQUIVALENT FLUID MODEL

In the acoustics of porous materials, one distinguishes two situations according to whether the frame is moving or not. In the first case, the dynamics of the waves due to the coupling between the solid skeleton and the fluid is well described by the Biot theory²⁴. In air-saturated porous media, the vibrations of the solid frame can often be neglected in absence of direct contact with the sound source, so that the waves can be considered to propagate only in fluid. This case is described by the equivalent-fluid model, which is a particular case of the Biot model, in which fluid-structure interactions are taken into account in two frequency response factors : dynamic tortuosity of the medium $\alpha(\omega)$ given by Johnson *et al*², and the dynamic compressibility of the air in the porous material $\beta(\omega)$ given by Allard *et al*¹. In the frequency domain, these factors multiply the density of the fluid and its compressibility respectively and represent the deviation from the behavior of the fluid in free space as the frequency increases. Consider a homogeneous porous material that occupies the region $0 \leq x \leq L$. A sound pulse impinges normally on the medium. It generates an acoustic pressure field p and an acoustic velocity field v within the material. The acoustic fields satisfy the following equivalent-fluid macroscopic equations (along the x -axis)¹ :

$$\rho\alpha(\omega)j\omega v = \frac{\partial p}{\partial x}, \quad \frac{\beta(\omega)}{K_a}j\omega p = \frac{\partial v}{\partial x}, \quad (1)$$

where, $j^2 = -1$, ρ is the saturating fluid density and K_a is the compressibility modulus of the fluid. In the low frequency domain, the viscous effects are important in all the pore volume, and the compression dilatation cycle in the porous material is slow enough to favor the thermal interactions between fluid and structure³. At the same time the temperature of the frame is practically unchanged by the passage of the sound wave because of the high value of its specific heat : the frame acts as a thermostat³. In addition, the thermal conductivity of the solid is high, and the excess heat is immediately evacuated by the solid, which therefore remains at the same temperature during the compression dilatation cycle³.

In the Darcy's regime^{25,26} (very low-frequency approximation), the expressions of the responses factors $\alpha(\omega)$ and $\beta(\omega)$ when $\omega \rightarrow 0$ are given by the relations²⁵ :

$$\alpha(\omega) = -\frac{\eta\phi}{\rho k_0 j\omega}, \quad \beta(\omega) = \gamma. \quad (2)$$

where k_0 is the static permeability, ϕ the porosity and γ the adiabatic constant.

The incident $p^i(t)$ and transmitted $p^t(t)$ fields are related in time domain by the transmission scattering operator^{19,20} T :

$$p^t(x, t) = \int_0^t T(\tau) p^i \left(t - \tau - \frac{(x - L)}{c_0} \right) d\tau. \quad (3)$$

The temporal operator kernel T is calculated by taking the inverse Fourier transform of the transmission coefficient of a slab of porous material given by (Appendix) :

$$\tilde{T}(\omega) = \frac{2Y(\omega)}{2Y(\omega) \cosh(jk(\omega)L) + (1 + Y^2(\omega)) \sinh(jk(\omega)L)}, \quad (4)$$

where :

$$Y(\omega) = \phi \sqrt{\frac{\beta(\omega)}{\alpha(\omega)}}, \quad \text{and} \quad k(\omega) = \omega \sqrt{\frac{\rho\alpha(\omega)\beta(\omega)}{K_a}}.$$

Using the expressions (2) of the dynamic tortuosity and compressibility, we obtain the following expression for the transmission coefficient :

$$\tilde{T}(\omega) = \frac{2C_1\sqrt{j\omega}}{2C_1\sqrt{j\omega} \cosh(LC_2\sqrt{j\omega}) + (1 + C_1^2 j\omega) \sinh(LC_2\sqrt{j\omega})}, \quad (5)$$

where

$$C_1 = \sqrt{\frac{\gamma\rho k_0\phi}{\eta}}, \quad C_2 = \sqrt{\frac{\gamma\eta\phi}{K_a k_0}} \quad (6)$$

By doing the Taylor series expansion of the transmission coefficient, when the frequency tends to zero ($\omega \rightarrow 0$), we obtain :

$$\tilde{T}(\omega) = \left(\frac{1}{1 + \frac{LC_2}{2C_1}} \right) \left[1 - \frac{LC_1 C_2 \left(1 + \frac{LC_2}{C_1} + \frac{1}{6} \left(\frac{LC_2}{C_1} \right)^2 \right)}{2 \left(1 + \frac{LC_2}{2C_1} \right)} j\omega + \dots \right] \quad (7)$$

As a first approximation, in the very low frequencies, the transmission coefficient is given by the first term :

$$\tilde{T} = \frac{1}{1 + \frac{LC_2}{2C_1}} = \frac{1}{1 + \frac{L}{2} \frac{\eta}{k_0 \sqrt{\rho K_a}}} \quad (8)$$

This simplified expression of the coefficient of transmission is independent of the frequency and the porosity of the material, and depends only on the static permeability and thickness of the material. In the next paragraph, we compare the expression (5) of the transmission coefficient, with its simplified expression (8), using numerical simulations of transmitted signals (Eq. 3) by a slab of air-saturated porous material.

Consider two samples M1 and M2 of air-saturated porous foams, having the same thickness $L = 0.05m$, and two different values of their static permeability. M1 is more permeable (less resistive) than M2. The international system of units for permeability is m^2 . A practical unit for permeability is the Darcy (D), ($1 \text{ Darcy} = 0.97 \times 10^{-12} m^2$). The permeability value of M1 is : $k_0 = 3092.8D$; (flow resistivity : $\sigma = 6000 \text{ N m}^{-4}s$), and of M2 ; $k_0 = 185.56D$ (flow resistivity : $\sigma = 10^5 \text{ N m}^{-4}s$) .

The incident signal and its spectrum are given in the figures (1-a) and (1-b), respectively. The frequency bandwidth of the incident signal is (450-550)Hz. Fig. 2 shows a comparison between two simulated transmitted signals computed with different expressions of the transmission coefficients for the sample M1. The first signal (solid line) corresponds to the simulated transmitted signal using the expression (5) of the transmission coefficient, and the second one (dashed line) using the relation (8). The amplitude is represented by an arbitrary unit and the point number represented in the abscissa is proportional to time. We note that for this frequency range (450-550)Hz, the transmitted waves predicted by the two terms of the transmission coefficient are slightly different, a small shift is observed between the two signals ; 10% for the amplitude, and 0.2% for the phase. By making the same comparison with the sample M2, which is less permeable than M1, the results given in figure 3 show a significant difference between the two

simulated signals (shift of 49% for the amplitude, and of 0.5% for phase). We can conclude that the approximation (8) of the transmission coefficient is much more accurate when the porous medium is more permeable (less resistive).

Another test is performed by taking an incident signal (Figs. 4-a, 4-b) with lower frequencies (30-70)Hz. The transmitted signals calculated from equations (5) and (8), are compared in figures 5 and 6, for the samples M1 and M2, respectively, in the frequency domain (30-70)Hz. These comparisons show a very good agreement, since it is practically impossible to distinguish between the two curves for both M1 and M2 samples. Indeed, the simplified expression of the transmission coefficient given by equation (8) is developed in very low frequencies. This study showed that the simplified expression (8) gives the same results as the expression (5) for the lower frequencies, especially for the most permeable (less resistive) materials. It would be more advantageous to use the simplified expression (Eq. 8) of the transmission coefficient, since it is fast, and does not depend on the frequency or porosity, and is simpler.

The transmission coefficient $\tilde{T}(\omega)$ given by Eq. 7 can be written as :

$$\tilde{T}(\omega) = \left(\frac{1}{1 + \frac{LC_2}{2C_1}} \right) \left[1 - j \frac{\omega}{\omega_c} \right], \quad (9)$$

where $\omega_c = \frac{2(1 + \frac{LC_2}{2C_1})}{LC_1C_2(1 + \frac{LC_2}{C_1} + \frac{1}{6}(\frac{LC_2}{C_1})^2)}$. The modulus of the transmission coefficient is given by :

$$|\tilde{T}(\omega)| = \left(\frac{1}{1 + \frac{LC_2}{2C_1}} \right) \sqrt{1 + \left(\frac{\omega}{\omega_c} \right)^2}.$$

Table (I) shows values of ω/ω_c and $|\tilde{T}(\omega)|$ for various values of frequency and flow resistivity.

It can be seen that the values of (ω/ω_c) are small compared to 1, and those of $|\tilde{T}(\omega)|$ are almost constant for the same value of the flow resistivity, especially for low frequencies and low resistivities. These results confirm those obtained in the previous paragraph, and again shows the possibility of using the simplified expression (8).

III. INVERSION OF EXPERIMENTAL DATA

The experimental set up is sketched in Fig. 7. Its picture is given in Fig. 8. The tube length is adaptable to avoid reflection, and to permit the propagation of transient signals, according to the frequency range desired. The tube material is PVC, its wall thickness is of 3 mm. For measurements in the frequency range (20-100)Hz, a length of 50m is sufficient. It is not important to keep the pipe straight ; it can be rolled in order to save space without perturbations on experimental signals. The tube diameter is 5 cm (the cut-off of tube $f_c \sim 4\text{kHz}$). A sound source Driver unit "Brand" constituted by loudspeaker Realistic 40-9000 is used. Tone-bursts are provided by synthesized function generator Stanford Research Systems model DS345-30MHz. The signals are amplified and filtered using model SR 650-Dual channel filter, Stanford Research Systems. The signals (incident and transmitted) are measured using the same microphone (Bruel&Kjaer, 4190) in the same position in the tube, avoiding the need of a calibration. The incident signal is measured without any porous sample, however, the transmitted signal is measured with the porous sample.

Consider a cylindrical sample of plastic foam F1 of diameter of 5 cm. Sample F1 was characterized using direct^{6,9} and indirect methods^{18,19}, given the values of the static viscous permeability and thickness marked by * in Table II. Fig. 9-a shows the experimental incident signal (dashed line) generated by the loudspeaker in the frequency bandwidth (20-40)Hz, and the experimental transmitted signal (solid line). Fig. 9-b shows their spectra. The inverse problem is to find the viscous static permeability k_0 and the thickness L of the porous sample, which minimize numerically the cost function $U(k_0, L) = \sum_{i=1}^{i=N} (p_{exp}^t(x, t_i) - p^t(x, t_i))^2$, wherein $p_{exp}^t(x, t_i)$ is the discrete set of values of the experimental transmitted signal and $p^t(x, t_i)$ the discrete set of values of the simulated transmitted signal. This inverse problem is solved numerically by the least-square method. For its iterative solution, we used the simplex search method (Nelder Mead)²⁷ which does not require numerical or analytic gradients. A large variation range is

applied for each estimated parameter in solving the inverse problem : $k_0 \in [1, 9] \times 10^3 D$ and $L \in [3, 7] \text{cm}$. The variations in the cost function present one clear minimum corresponding to the following solution of the inverse problem : $k_0 = 560D$, or $\sigma = 33137 \text{ N m}^{-4}\text{s}$, and $L = 2.49\text{cm}$. Using these values, we present in Figs. 10(a)-10(b), the variations of the cost function U when varying only one of the parameters around the minimum. This result is consistent with what has been found using classical methods^{6,9,18,19} (marked by * in Table II). A comparison between an experimental transmitted signal and simulated transmitted signal is given in Fig. 11 for the optimized values of the inverted parameters. The agreement between theoretical and experimental data is good, which leads us to conclude that the optimized values of the viscous permeability and the thickness of the sample are accurate within 10%. This study has been also carried, out in other frequency bandwidth summarized in Table II. The inverse problem has also been solved if the material thickness is held constant, the results of the inversion are marked by ‡. It can be seen that for the different frequency bandwidths of the experimental incident signals, the optimized values obtained using this method are close to those produced using classical methods^{6,9,18,19}. The results of the inversion for the permeability when the thickness is known or unknown are slightly different but close enough in general. Two other plastic foams samples F2 and F3, having a very different values of their permeability are also studied. Their characteristics^{6,9,18,19} are marked by * in Tables III and IV. After solving the inverse problem in different frequency regimes, the results are presented in the same Tables (III and IV). Note that for the sample F3, which is highly permeable (not resistive), it was possible to use experimental data at relatively high frequency (up to 1kHz). In this case, the approximation (8) remains valid for higher frequencies, relative to samples F1 and F2, which are much less permeable. Here, again the inverted values of the permeability of the samples F2 and F3, are very close to those given by the classical methods^{6,9,18,19} (marked by * in Tables III and IV). The difference between the optimized values and those given by other methods^{6,9,18,19} does not exceed 10%, except for the inverted value of the thickness (sample F2) in the frequency band (20-40)Hz.

The results of the inverse problem (Tables II, III and IV) show that overall (except for some cases) the results of the inversion for the permeabilities are better when the material thickness is assumed unknown. The advantage of solving the inverse problem with respect to the thickness, is to check the results of the inversion, since the thickness of the material is easily measurable. This simple method seems to be effective for measuring the permeability of the porous material saturated with air and offers another faster and simpler alternative to conventional methods^{6–23}. Note that this experiment can not be used for very highly resistive porous materials (permeability $< 9 \times 10^{-11} \text{m}^2$). In fact, the acoustic excitation generates Biot's vibrations of the structure²⁷, which induces a structural disturbance resulting from the elasticity. These vibrations are not taken into account in the equivalent fluid model used in this work.

The great advantage of this method compared to other indirect methods using acoustic waves^{12–23}, is that it is not necessary, to know in advance, the value of the porosity, in order to measure the permeability of the porous material.

*

IV. CONCLUSION

A simple and effective method is developed for the experimental measurement of the static viscous permeability (or flow resistivity) and thickness of an air-saturated porous material. The development of the transmission coefficient in the Darcy's regime (low frequency), was used to extract a simplified expression. This study shows that this new expression gives the same results as the general one dependent on the frequency^{19,20}, but has the advantage of being more reliable, simpler and faster. The inverse problem is solved using experimental transmitted data. The reconstructed values of permeability and thickness are close to those using classical methods^{7–9,11}. The most important result in this study is that it is now possible to measure the viscous permeability and thickness, without knowing the porosity of the materials, and just

by using the experimental transmitted wave. This method is an improved alternative to other methods using acoustic methods^{7–9,17–21}, or non-acoustic methods^{10–16}.

APPENDIX A : REFLECTION AND TRANSMISSION COEFFICIENTS

Let us consider a homogeneous, isotropic and rigid porous material structure, which occupies a finite space, $0 \leq x \leq L$. A short sound pulse arrives at $x = 0$ at normal incidence on the material at time $t = 0$. It gives rise to an acoustic pressure field $p(x)$ and an acoustic velocity field $v(x)$ within the material, which satisfying the equations (1). The expression of a pressure wave incident plane, unit amplitude, arriving at normal incidence to the porous material is given by :

$$p^i(x, t) = e^{-j(k_0x - \omega t)}, \quad \text{where} \quad k_0 = \frac{\omega}{c_0} = \omega \sqrt{\frac{\rho}{K_a}},$$

k_0 is the wave number of the free fluid. In the medium (I) ($x < 0$), the movement results from the superposition of incident and reflected waves :

$$p_1(x, t) = e^{-j(k_0x - \omega t)} + R e^{-j(-k_0x - \omega t)}, \quad (10)$$

where R is the reflection coefficient. According to Eq. 1, the expression of the velocity field in the medium (I) wrote :

$$v_1(x, t) = \frac{1}{Z_f} \left(e^{-j(k_0x - \omega t)} - R(\omega) e^{-j(-k_0x - \omega t)} \right), \quad \text{where} \quad Z_f = \sqrt{\rho K_a}. \quad (11)$$

In the medium (II) corresponding to the porous material, the expressions of the pressure and velocity fields are :

$$p_2(x, t) = A(\omega) e^{-j(k(\omega)x - \omega t)} + B(\omega) e^{-j(-k(\omega)x - \omega t)}, \quad (12)$$

$$v_2(x, t) = \frac{1}{Z(\omega)} \left(A(\omega) e^{-j(k(\omega)x - \omega t)} - B(\omega) e^{-j(-k(\omega)x - \omega t)} \right) \quad (13)$$

In these expressions $A(\omega)$ and $B(\omega)$ are function of pulsation for determining, $Z(\omega)$ and $k(\omega)$ are the characteristic impedance and the wave number, respectively, of the acoustic wave in the porous medium. These are two complex quantities :

$$k(\omega) = \omega \sqrt{\frac{\rho \alpha(\omega) \beta(\omega)}{K_a}}, \quad Z(\omega) = \sqrt{\frac{\rho K_a \alpha(\omega)}{\beta(\omega)}}$$

Finally, in the medium (III), the expressions of the pressure and velocity fields of the wave transmitted through the porous material are :

$$p_3(x, t) = \tilde{T}(\omega) e^{-j(k_0(x-L) - \omega t)}, \quad (14)$$

$$v_3(x, t) = \frac{1}{Z_f} \tilde{T}(\omega) e^{-j(k_0(x-L) - \omega t)}, \quad (15)$$

where $\tilde{T}(\omega)$ is the transmission coefficient. To determine the coefficients of reflection and transmission, the continuity conditions are written to the interfaces $x = 0$ and $x = L$ of the porous medium :

$$p_1(0^-) = p_2(0^+), \quad p_2(L^-) = p_3(L^+), \quad v_1(0^-) = \phi v_2(0^+), \quad \phi v_2(L^-) = v_3(L^+). \quad (16)$$

Using expressions (16) and relations (10-15), we obtain the following relations between $A(\omega)$, $B(\omega)$, $R(\omega)$ and $T(\omega)$:

$$\begin{aligned} 1 + R(\omega) &= A(\omega) + B(\omega), \quad \tilde{T} = A(\omega) e^{-jk(\omega)L} + B(\omega) e^{jk(\omega)L}, \\ 1 - R(\omega) &= \phi \frac{Z_f}{Z(\omega)} (A(\omega) - B(\omega)), \quad \tilde{T}(\omega) = \phi \frac{Z_f}{Z(\omega)} (A(\omega) e^{-jk(\omega)L} - B(\omega) e^{jk(\omega)L}). \end{aligned}$$

The solution of this system of equations gives the expressions of the reflection and transmission coefficients :

$$\begin{aligned} R(\omega) &= \frac{(1 - Y^2(\omega))}{(1 + Y^2(\omega)) + 2Y(\omega) \coth(jk(\omega)L)} \\ \tilde{T}(\omega) &= \frac{2Y(\omega)}{2Y(\omega) \cosh(jk(\omega)L) + (1 + Y^2(\omega)) \sinh(jk(\omega)L)}, \end{aligned}$$

where :

$$Y(\omega) = \phi \sqrt{\frac{\beta(\omega)}{\alpha(\omega)}}.$$

APPENDIX B : TAYLOR EXPANSION OF THE TRANSMISSION COEFFICIENT

: The transmission coefficient $\tilde{T}(\omega)$ is given by (5) :

$$\begin{aligned}\tilde{T}(\omega) &= \frac{2C_1\sqrt{j\omega}}{2C_1\sqrt{j\omega}\cosh(LC_2\sqrt{j\omega}) + (1 + C_1^2j\omega)\sinh(LC_2\sqrt{j\omega})}, \\ &= \left(\frac{1}{\cosh(LC_2\sqrt{j\omega})}\right) \left[\frac{1}{1 + \frac{(1+C_1^2j\omega)}{2C_1\sqrt{j\omega}}\tanh(LC_2\sqrt{j\omega})}\right]\end{aligned}$$

The Taylor series expansion in the vicinity of zero, of $\cosh(LC_2\sqrt{j\omega})$ and $\tanh(LC_2\sqrt{j\omega})$, gives :

$$\begin{aligned}\cosh(LC_2\sqrt{j\omega}) &= 1 + \frac{1}{2}L^2C_2^2j\omega + O([j\omega]^2) \\ \frac{1}{\cosh(LC_2\sqrt{j\omega})} &= 1 - \frac{1}{2}L^2C_2^2j\omega + O([j\omega]^2),\end{aligned}\tag{17}$$

and

$$\tanh(LC_2\sqrt{j\omega}) = LC_2\sqrt{j\omega} - \frac{1}{3}L^3C_2^3(j\omega)^{3/2} + O(j\omega)^{5/2}.$$

We have

$$\left(\frac{1 + C_1^2j\omega}{2C_1\sqrt{j\omega}}\right)\tanh(LC_2\sqrt{j\omega}) = \frac{1}{2}\frac{LC_2}{C_1} + \frac{1}{2C_1}\left(-\frac{1}{3}L^3C_2^3 + LC_2C_1^2\right)j\omega + O([j\omega]^2)$$

and

$$\left[\frac{1}{1 + \left(\frac{1+C_1^2j\omega}{2C_1\sqrt{j\omega}}\right)\tanh(LC_2\sqrt{j\omega})}\right] = \left(\frac{1}{1 + \frac{LC_2}{2C_1}}\right)\left(1 - \frac{\left(-\frac{1}{3}L^3C_2^3 + LC_2C_1^2\right)j\omega + O([j\omega]^2)}{2C_1\left(1 + \frac{LC_2}{2C_1}\right)}\right)\tag{18}$$

Using Eqs. (17) and (18), one finds

$$\tilde{T}(\omega) = \left(\frac{1}{1 + \frac{LC_2}{2C_1}}\right)\left[1 - \left(\frac{L^2C_2^2}{2} + \frac{\left(-\frac{1}{3}L^3C_2^3 + LC_2C_1^2\right)}{2C_1\left(1 + \frac{LC_2}{2C_1}\right)}\right)j\omega + O([j\omega]^2)\right]$$

Simplifying and rearranging terms, we find Eq. 7 :

$$\tilde{T}(\omega) = \left(\frac{1}{1 + \frac{LC_2}{2C_1}}\right)\left[1 - \frac{LC_1C_2\left(1 + \frac{LC_2}{C_1} + \frac{1}{6}\left(\frac{LC_2}{C_1}\right)^2\right)}{2\left(1 + \frac{LC_2}{2C_1}\right)}j\omega + O([j\omega]^2)\right]$$

Table I. Values of (ω/ω_c) and $|\tilde{T}(\omega)|$ for different values of frequency and resistivity.

	50 Hz		100 Hz		300 Hz		500 Hz		1000 Hz	
σ ($Nm^{-4}s$)	ω/ω_c	$ \tilde{T} $	ω/ω_c	$ \tilde{T} $	ω/ω_c	$ \tilde{T} $	ω/ω_c	$ \tilde{T} $	ω/ω_c	$ \tilde{T} $
3000	0.03	0.84	0.06	0.84	0.2	0.86	0.34	0.89	0.68	1.02
10^4	0.04	0.62	0.08	0.62	0.26	0.64	0.44	0.68	0.88	0.83
10^5	0.15	0.14	0.31	0.14	0.93	0.19	1.55	0.26	3.1	0.46
10^6	1.21	0.02	2.43	0.04	7.29	0.12	12.16	0.19	24.32	0.39

Table II. Characteristics of the sample F1 obtained by conventional methods (marked by *), either by solving the inverse problem only for the permeability (marked by #), or by solving the inverse problem simultaneously for the permeability and thickness (not marked).

Frequency range (Hz)	(20-40)	(30-50)	(40-60)	(200-600)	
Permeability k_0 (<i>Darcy</i>)	559 [#]	592 [#]	513 [#]	604 [#]	540*
Flow resistivity σ (N m ⁻⁴ s)	31304 [#]	31346 [#]	36145 [#]	30717 [#]	34364*
Permeability k_0 (<i>Darcy</i>)	560	580	505	553	540*
Flow resistivity σ (N m ⁻⁴ s)	33137	31994	36746	33556	34364*
Thickness L (cm)	2.49	2.45	2.45	2.34	2.5*

Table III. Characteristics of the sample F2 obtained by conventional methods (marked by *), either by solving the inverse problem only for the permeability (marked by #), or by solving the inverse problem simultaneously for the permeability and thickness (not marked).

Frequency range (Hz)	(20-40)	(30-50)	(40-60)	(50-70)	(60-80)	
Permeability k_0 (<i>Darcy</i>)	180 [#]	228 [#]	267 [#]	225 [#]	197	226*
Flow resistivity σ (N m ⁻⁴ s)	102860 [#]	81448 [#]	69498 [#]	82569 [#]	94241 [#]	81820*
Permeability k_0 (<i>Darcy</i>)	211	232	231	231	222	226*
Flow resistivity σ (N m ⁻⁴ s)	87946	79986	80332	80332	83589	81820*
Thickness L (cm)	4.8	4.24	4.16	4.06	4.6	4.15*

Table IV. Characteristics of the sample F3 obtained by conventional methods (marked by *), either by solving the inverse problem only for the permeability (marked by #), or by solving the inverse problem simultaneously for the permeability and thickness (not marked).

Frequency range (Hz)	(30-50)	(300-500)	(500-700)	(600-800)	(800-1000)	
Permeability k_0 (<i>Darcy</i>)	2515.4 [#]	3309.3 [#]	3226,8 [#]	2546.4 [#]	4505.2 [#]	2886*
Flow resistivity σ (N m ⁻⁴ s)	7377 [#]	5607.5 [#]	5750.8 [#]	5187.3 [#]	4119 [#]	6428*
Permeability k_0 (<i>Darcy</i>)	2783	3144	3144	2938	2938	2886*
Flow resistivity σ (N m ⁻⁴ s)	6668	5902	5902	6316	6316	6428*
Thickness L (cm)	5.5	4.92	5.3	5.08	5.2	5*

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FIGURE CAPTIONS

Fig. 1-a. Incident signal (450-550)Hz.

Fig. 1-b. Spectrum of the incident signal (450-550)Hz.

Fig. 2. (Color online) Comparison between simulated transmitted signals corresponding to Eq.(5) (solid line) and Eq.(8) (dashed line), for the sample M1.

Fig. 3. (Color online) Comparison between simulated transmitted signals corresponding to Eq. (5) (solid line) and Eq. (8) (dashed line), for the sample M2.

Fig. 4-a. Incident signal (30-70)Hz.

Fig. 4-b. Spectrum of the incident signal (30-70)Hz.

Fig. 5. (Color online) Comparison between simulated transmitted signals corresponding to Eq. (5) (solid line) and Eq. (8) (dashed line), for the sample M1, in the frequency band (30-70)Hz.

Fig. 6. (Color online) Comparison between simulated transmitted signals corresponding to Eq. (5) (solid line) and Eq. (8) (dashed line), for the sample M2, in the frequency band (30-70)Hz.

Fig. 7. Experimental setup of acoustic measurements.

Fig. 8. (Color online) Picture of the experimental setup.

Fig. 9-a. Experimental incident signal (dashed line) and experimental transmitted signal (solid line).

Fig. 9-b. Spectra of the incident (dashed line) and experimental transmitted signal (solid line).

Fig. 10-a. Variation of the minimization function U with static viscous permeability k_0 .

Fig. 10-b. Variation of the minimization function U with the thickness of the porous material L .

Fig. 11. (Color online) Comparison between the experimental transmitted signal (dashed line) and the simulated transmitted signals (solid line) using the reconstructed values of k_0 and L .